Using Alternative Spatial data sources for SAE in developing countries

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Small area estimation problem

- Interest on estimation of social and demographic indicators at sub-national levels
- Estimates are often built using survey data. Insufficient sample sizes impede obtaining reliable estimates for small areas under the typical survey inference framework
- Area-specific indicator is seen as a realisation of a statistical model with common terms across areas. Use of auxiliary data sources and statistical models to *borrow strength* across areas
- Very active area of research. See Rao and Molina (2015); Pfeffermann (2013).

Alternative data sources

$$\text{Survey} \sim \text{Census} + \text{Administrative records} + \ldots$$

- Remotely Sensed (RS) data
- Mobile phone (CDR) data
- Web-scraped data, Social media data, ...

Possible uses as covariates, response, reference,...

See Marchetti et al. (2015); MAKSWELL Project, Work Package 2: van den Brakel, Buelens, et al. (2019) and van den Brakel, Smith, et al. (2019).

Use of RS data for SAE

Advantages

- Broadly available and frequently updated (*no one is left behind*). Low cost. Particularly useful in low-income countries where high quality survey, census and administrative data may be scarce. See:
  - US geological survey `https://earthexplorer.usgs.gov/`
  - European Space Agency `www.geoportal.org`
  - `http://trends.earth/docs/en/` (land coverage)
  - Open Street Map `http://extract.bbbike.org/`
  - WorldPop Research Project `http://www.worldpop.org.uk/`
  - ... (more at the end).
- Flexible definition of target geography

Use of RS data for SAE

Limitations

- Explanatory power? Unclear link between variables and outcome
- Potential for irregular coverage, e.g., due to atmospheric conditions
- Substantial pre-processing. Measurement error?
- Potential for uneven reference periods
- It is not clear in which situations a given aggregation strategy should be preferred
- Adequate use requires some degree of specialized knowledge

Modelling approaches

Statistical vs Algorithmic/Mapping approaches †.

Statistical modelling (SAE)

- Real observations of the phenomenon are required (survey)
- Sampling design is taken into account
- Model assessment (GOF) and area-specific uncertainty of estimates (MSE)
- Area-level models: Fay-Herriot: Frequentist/HB; can include spatial/temporal effects
- Relatively coarse geography. Administrative boundaries + survey design

† For a detailed discussion from a geospatial perspective see the report from the Task Team on Satellite Imagery and Geospatial data, UN (2017)

Modelling approaches (2)

Algorithmic/Mapping approaches

- Observations required (supervised methods)
- Generally, no consideration of sampling design
- GLMM's, classification trees, support vector machines,... Often inclusion of spatial effects as default
- Aim to very granular geographies
- Crossvalidation. MCR and MSEP. If Bayesian inference, uncertainty measured using posterior variances
- Examples:
  - Poverty measurement in Bangladesh; Steele et al. (2017)
  - Slum mapping in Casablanca, Morocco; Rhinane et al. (2011)

Presentation aims

Illustration of both approaches in a realistic set-up.
Poverty measurement in Bangladesh in the spirit of Steele et al. (2017). Wealth index $\sim$ RS covariates.

Aims:

- Identify common points and differences between both approaches
- Illustrate the use of standard packages for each approach (`sae`, `BRugs`, `inla`) for the fitting of spatial and non-spatial models
- Identify potential methodological issues

Poverty measurement in Bangladesh

Target: Average WI by Upazila (Level 3).

Survey data - DHS 2014

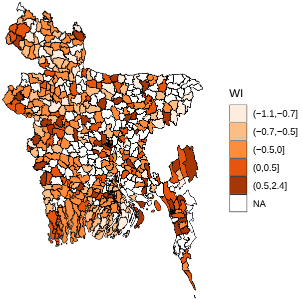
- Stratified 2-stage cluster design. At least one cluster selected in 365/508 (72%) Upazilas
- Response: WI computed via PCA
- $n = 17$K households. $\bar{n}_i = 34$ households

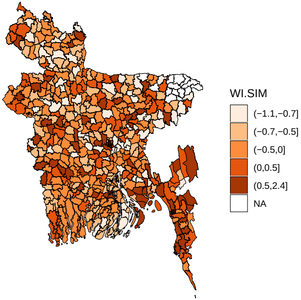
RS data:

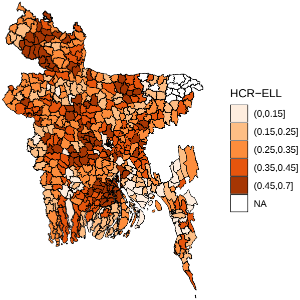
- 18 variables as starting point in Steele et al. (2017).
  - Night time lights (NOAA-US)
  - Elevation (CGIAR-CSI)
  - Accessibility to areas with more than 50K people (A global map of accessibility - European Commission Joint Research Centre)

Complete cases exercise

Imputation of direct estimates and their sampling variances in out-of sample areas.







Methods

Fay-Herriot model

$$\boxed{\hat{\theta}_i^d = \boldsymbol{x}_i^T \boldsymbol{\beta} + u_i + e_i}$$

$\hat{\theta}_i^d$ a direct estimator of θ_i; $u_i \overset{iid}{\sim} N(0, \sigma_u^2)$; $e_i \overset{ind}{\sim} N(0, \sigma_i^2)$.

From a frequentist perspective, σ_i^2 is assumed known (estimation + smoothing). An EBLUP for θ is given by

$$\hat{\theta}_i^{FH} = \boldsymbol{x}_i^T \hat{\boldsymbol{\beta}} + \hat{u}_i = \hat{\gamma}_i \hat{\theta}_i^d + (1 - \hat{\gamma}_i) \boldsymbol{x}_i^T \hat{\boldsymbol{\beta}}$$

with $\hat{\gamma}_i = \hat{\sigma}_u^2 / (\sigma_i^2 + \hat{\sigma}_u^2)$. Analytic MSE estimator by Prasad and Rao, 1990. Parametric bootstrap can also be used. Available in R package `sae`.

Methods

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From a Bayesian (HB) perspective:

$$\hat{\theta}_i^d | \theta_i \overset{ind}{\sim} N(\theta_i, \sigma_i^2)$$

$$\theta_i | \beta, \sigma_u^2 \overset{ind}{\sim} N(\boldsymbol{x}_i^T \boldsymbol{\beta}, \sigma_u^2)$$

Information on σ_i^2 can included via another level or an informative prior. See You and Chapman (2006).

The posterior mean and variance of θ_i are used for inference.

Available in R packages `BayesSAE`, `hbsae`. Can also use BUGS, JAGS, Stan, ...

Methods (2)

Fay-Herriot model - Spatial extensions

SAR

Assumes $\mathbf{v} = \rho \mathbf{W}\mathbf{v} + \mathbf{u}$, with $\mathbf{u} \sim N(\mathbf{0}, \sigma_u^2 \mathbf{I})$, therefore

$$\hat{\boldsymbol{\theta}}^d = \mathbf{X}\boldsymbol{\beta} + (\mathbf{I} - \rho \mathbf{W})^{-1}\mathbf{u} + \mathbf{e}$$

See Cressie (1993) and Pratesi and Nicola Salvati (2008).

- $\mathbf{W}$ is an adjacency matrix row-standardised to sum 1, leading to $\rho \in (-1, 1)$.
- ML/REML estimates obtained via `sae`. MSE analytical + bootstrap.

Methods (2)

Fay-Herriot model - Spatial extensions

iCAR

Besag, York, and Mollié (1991). It assumes

$$v_i | v_{j \neq i}, \sigma_u^2 \sim N\left(\frac{1}{n_i}\sum_{j \sim i} v_j,\ \frac{1}{n_i}\sigma_u^2\right) \tag{1}$$

which leads to $\mathbf{v} \sim N(\mathbf{0}, \sigma_u^2(\mathbf{D} - \mathbf{W}))$, with $\mathbf{D} = diag\,\{n_i\}$ and $\mathbf{W}$ an adjacency matrix. Notice that (1) doesn't define a proper distribution as $(\mathbf{D} - \mathbf{W})$ is non-invertible. The constraint $\sum_i v_i = 0$ is required to make the v_i identifiable.

- Computationally convenient as covariance matrix is sparse
- Available in R-INLA: latent specification `besag`, `bym`, and in BUGS: distribution `car.normal`.

Fitting non-spatial models

Complete cases. Point estimates and variances obtained using the sampling design. Smoothing of variance estimates using GVF.

$$\widehat{WI_i} = \beta_0 + \beta_1 \times ELEV + \beta_2 \times NL + \beta_3 \times ACC + u_i + e_i$$

$u_i \overset{iid}{\sim} N(0, \sigma_u^2)$; $e_i \overset{ind}{\sim} N(0, \sigma_i^2)$. R-INLA latent specification `iid`.

- M1 Standard FH model using `sae`. $\sigma_i^2 = \hat{\sigma}_i^2$ fixed
- M2 Standard Gaussian model in R-INLA. $\sigma_i^2 = \sigma_e^2$ unknown
- M3 R-INLA with $\sigma_{e_i}^2 = g_i \sigma_e^2$; $g_i = v_i / \bar{v}_i$ fixed.
  $\tau = 1/\sigma_e^2$; $\pi(\tau) \sim \text{Gamma}\left(\frac{\bar{n}_i - 1}{2} - 1\,,\ \frac{(\bar{n}_i - 1)\bar{v}_i}{2}\right)$.
- M4 HB using `BRugs` with $\pi(\tau_i)$ as in M3.

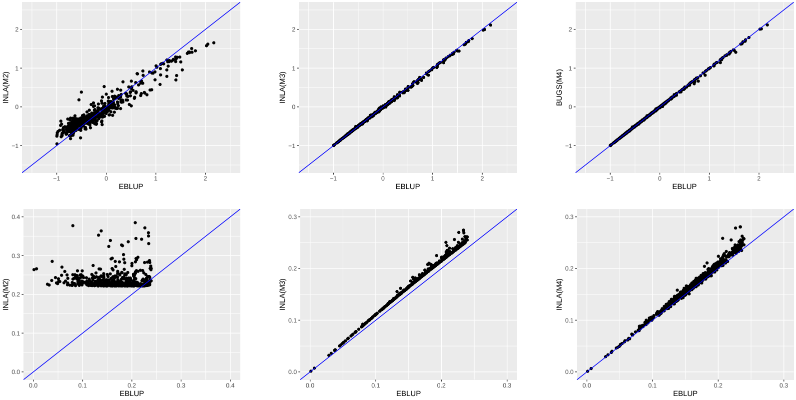
Results non-spatial models

- Small differences in the fixed effects
- Large differences in the variance decomposition. Using scaling to allow for heteroscedasticity nearly eliminates all differences

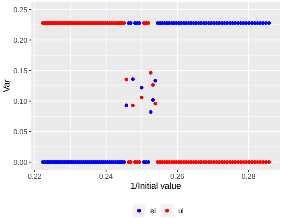
	M1	M2	M3	M4
$\hat{\beta}_0$	0.6662	0.6866	0.6604	0.6469
$\hat{\beta}_{elev}$	-0.0553	-0.0530	-0.0557	-0.0548
$\hat{\beta}_{nl}$	0.3137	0.3112	0.3141	0.3159
$\hat{\beta}_{acc}$	-0.0878	-0.0892	-0.0874	-0.0838
$\hat{\sigma}_e^2$	0.0362	0.1219	0.0423	0.0408
$\hat{\sigma}_u^2$	0.1889	0.1058	0.1800	0.1838

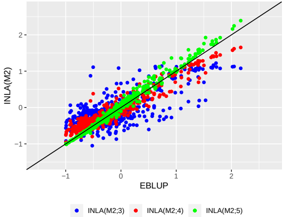
Results non-spatial models (2)

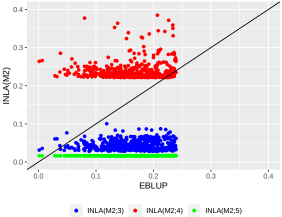
- Some impact on the point estimates
- Large Impact on uncertainty measures



Results non-spatial models (3)







Fitting spatial models

- M5 SAR correlation in `sae`. $\sigma_i^2 = \hat{\sigma}_i^2$ fixed
- M6 iCAR correlation in R-INLA, `besag` specification. $\sigma_i^2 = \sigma_e^2$ unknown
- M7 As M6, `bym` specification.
- M8 As M7, $\sigma_{e_i}^2 = g_i \sigma_e^2$; $g_i = v_i / \bar{v}_i$ fixed.
  $\tau = 1/\sigma_e^2;\ \pi(\tau) \sim \text{Gamma}\left(\frac{\bar{n}_i - 1}{2} - 1\,,\ \frac{(\bar{n}_i - 1)\bar{v}_i}{2}\right)$.
- M9 HB using `BRugs` with $\pi(\tau_i)$ as in M8.

Results spatial models

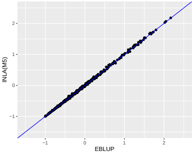
	M1	M5	M6	M7	M8	M9
$\hat{\beta}_0$	0.6662	0.6567	0.4208	0.4141	0.4147	0.3414
$\hat{\beta}_{elev}$	-0.0553	-0.0472	0.0688	0.0718	0.0716	0.0869
$\hat{\beta}_{nl}$	0.3137	0.2999	0.2714	0.2695	0.2695	0.2649
$\hat{\beta}_{acc}$	-0.0878	-0.0939	-0.1086	-0.1093	-0.1093	-0.1078
$\hat{\sigma}_e^2$	0.0362	0.0362	0.1875	0.0793	0.0378	0.0408
$\hat{\sigma}_u^2$	0.1889	0.1816	0.0432	0.1020	0.1480	0.1344
$\hat{\sigma}_v^2$				0.0448	0.0437	0.1144
ρ		0.2458				

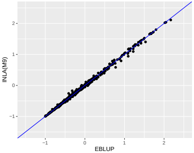
- Spatial confounding for the iCAR models. See Reich, Hodges, and Zadnik (2006), Prates, Assunção, Rodrigues, et al. (2019)

Results spatial models (2)

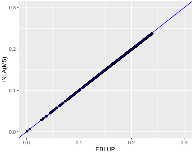
M5 (SAR - `sae`) and M9 (iCAR - `BRugs`):

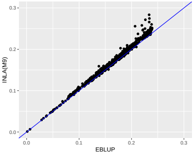
$\widehat{WI}_i$





SRMSE



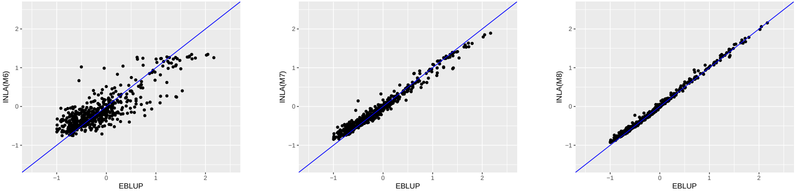


- Not relevant improvement by the inclusion of spatially correlated random effects.

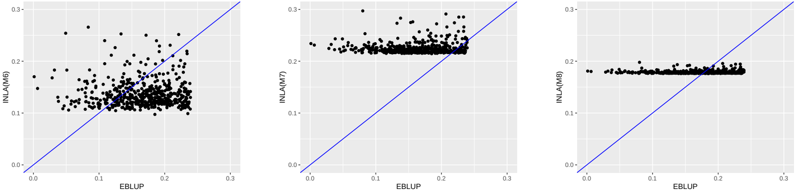
Results spatial models (3)

M6 (Besag), M7 (bym) and M8 (bym + scale) - `R-INLA`:

$\widehat{WI_i}$



SRMSE



Concluding remarks

- Ready to use software for approximate Bayesian inference offers interesting possibilities. However, some degree of specialized knowledge is necessary for its correct use.
- The use of spatially correlated effects did not improve substantially our estimates. Spatial confounding appears under iCAR. Negligible effects on the point/variance estimates.
- Although the proposed specification of models for R-INLA leads to reasonable point estimates, more work is necessary to identify a way to produce appropriate variance estimates.
- Including spatially correlated random effects may help to improve estimates in out-of sample areas. However, assessing whether the type of shrinkage induced by a particular model is reasonable for SAE purposes would be relevant. Gomez-Rubio et al. (2005), Saei and Chambers (2005), Chandra, N. Salvati, and Chambers (2007).

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- European Space Agency www.geoportal.org
- http://trends.earth/docs/en/ (land coverage)
- Open Street Map http://extract.bbbike.org/
- WorldPop Research Project http://www.worldpop.org.uk/
- National Oceanic and Atmospheric Administration http://ngdc.noaa.gov/eog/viirs.html
- European Commission Joint Research Centre https://forobs.jrc.ec.europa.eu/products/gam/
- Center for International Earth Science Information Network(CIESIN) http://sedac.ciesin.columbia.edu/data/
- CGIAR Consortium for Spatial Information http://www.cgiar-csi.org/data
- WorldClim Global Climate Data http://www.worldclim.org
- European Commission Global Human Settlement Layer https://ghsl.jrc.ec.europa.eu/
- World Database on Protected Areas http://www.protectedplanet.net/
- Oak Ridge National Laboratory Land Coverage http://webmap.ornl.gov/wcsdown/wcsdown.jsp?dg_id=10024_1
- ETH Zurich International Conflict Research http://www.icr.ethz.ch/data/geoepr